Hand-In Exercise: State Estimation

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1 System Description

This section presents the considered model and state estimator for discrete time linear time invariant systems that are affected by process noise and measurement noise. We consider the following discrete time system

$$\begin{aligned} x_{k+1} &= Ax_k + Gw_k, \\ y_k &= Cx_k + v_k, \end{aligned} \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$ is the system matrix, $G \in \mathbb{R}^{n\times m}$ is the noise input matrix, $C \in \mathbb{R}^{p\times n}$ is the output matrix, $x_k \in \mathbb{R}^n$ is the state at sample k, $y_k \in \mathbb{R}^p$ is the measurement at sample k, $w_k \in \mathbb{R}^m$ is the process noise at sample k, and $v_k \in \mathbb{R}^p$ is the measurement noise at sample k. The numerical values of system matrices are

$$A = \begin{bmatrix} 0.1 & 0 & 0.1 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.3 \end{bmatrix}, \quad C = \begin{bmatrix} 0.1 & 0.2 & 0 \end{bmatrix}, \quad G = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}. \tag{2}$$

The difference equation of the considered state estimator (Kalman filter) is

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k} + Bu_k \tag{3}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + L(y_k - C\hat{x}_{k|k-1}) \tag{4}$$

where L is the stabilizing gain of dimension 3×1, and $\hat{x}$ is the estimated state.

2 Initial Kalman Filter Design

This section presents the design of an initial Kalman filter and an evaluation of its properties. The data file `sim_data.csv` contains data from 200 simulations of (1). It contains three columns: 'experiment', 'time', and 'y'. The data from experiment 1 is shown in Figure 1.

The Kalman gain is computed in MATLAB using the following command

$$L = \text{dlqe}(A, G, C, Q_w, R_v) \tag{5}$$

where the numerical values of Qw = 0.9 and Rv = 0.5

The designed Kalman filter is validated by applying it to the data given in Figure 2. The residual vector is given in Figure 3 and the autocovariance is illustrated in Figure 4.

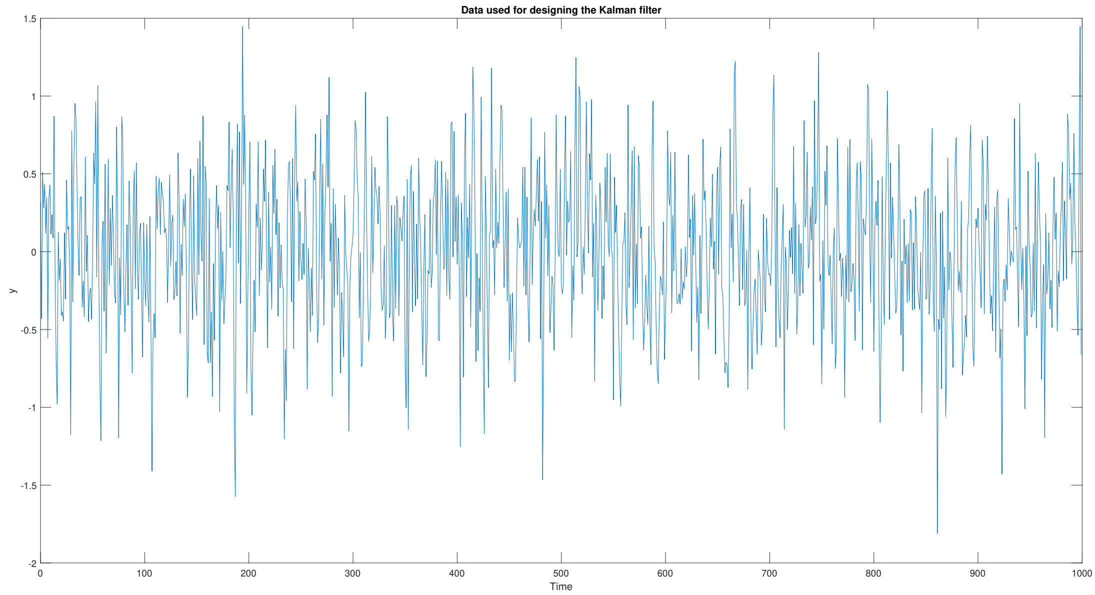

Figure 1: Data used for designing the Kalman filter.

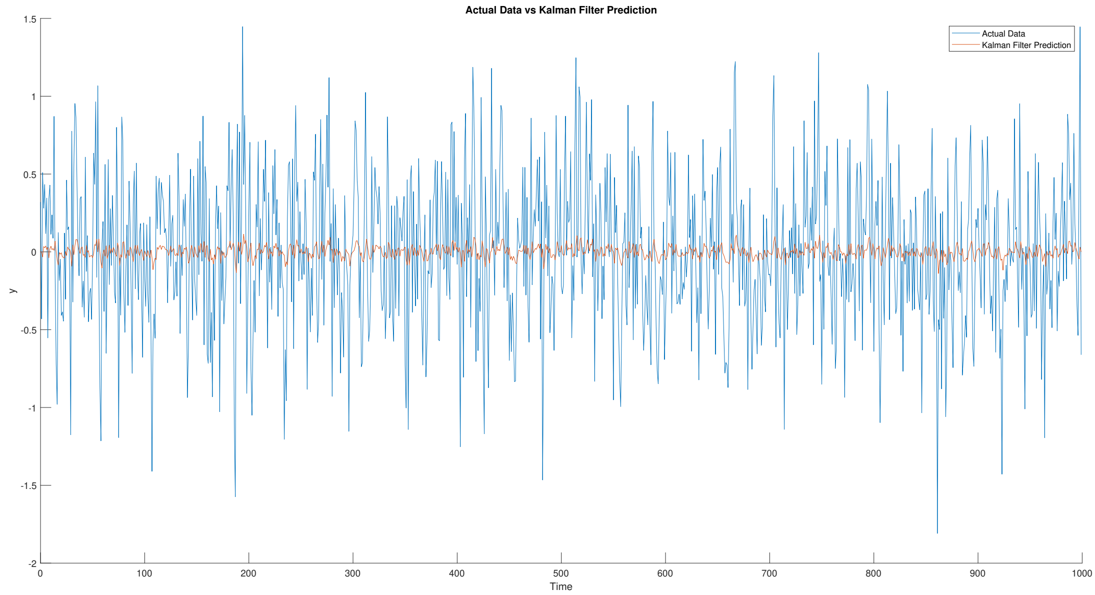

Figure 2: Data used for validation of the Kalman filter.

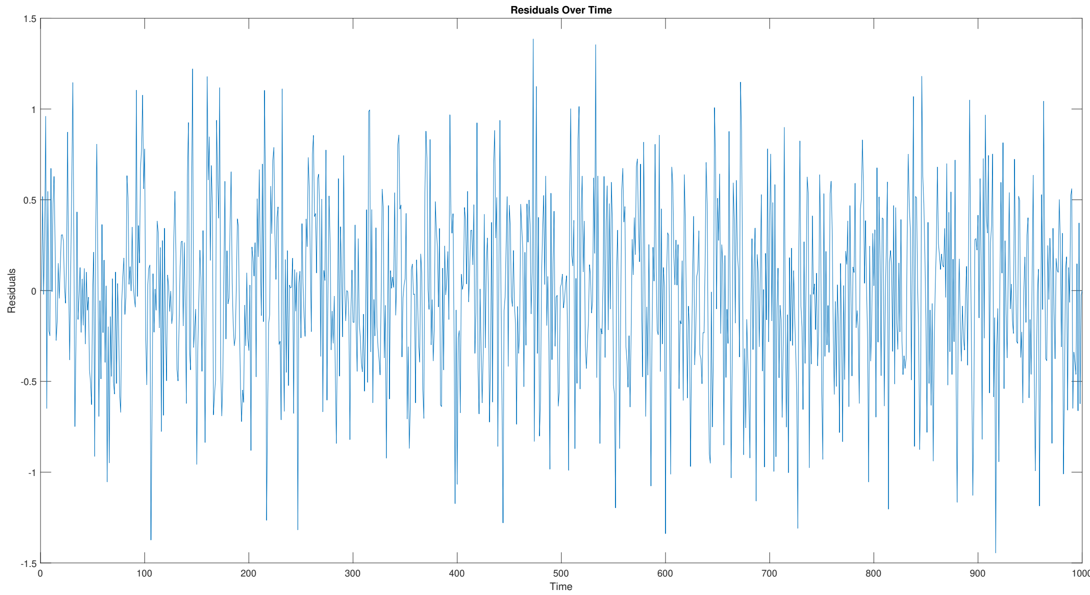

Figure 3: Residual of designed Kalman filter.

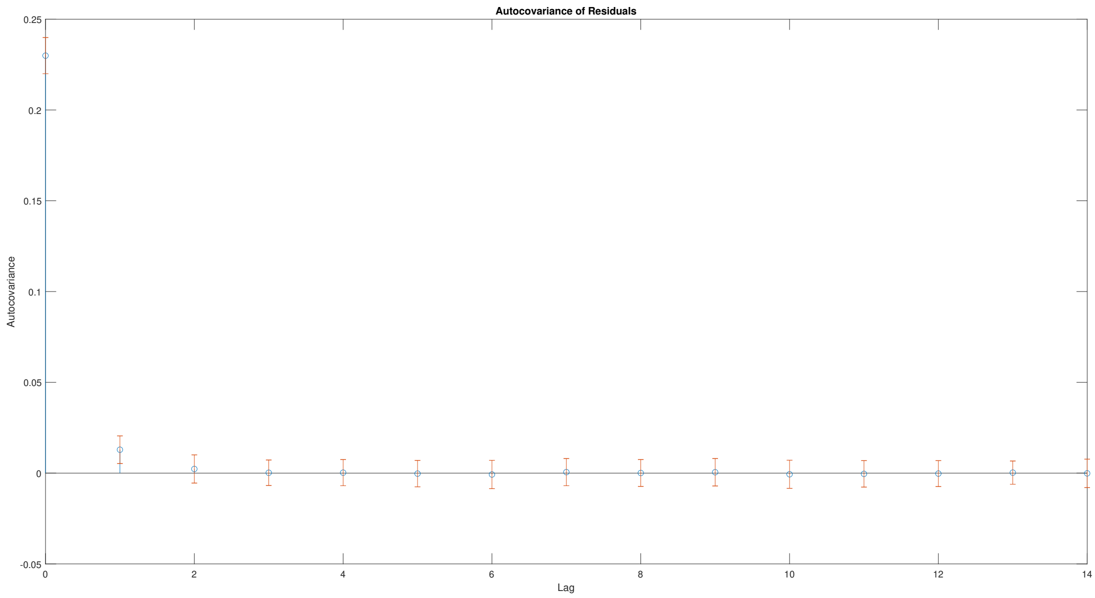

Figure 4: Autocovariance of designed Kalman filter with lag up to 15 samples.

3 Tuning of Kalman Filter

This section describes the results of following the autocovariance least-squares estimation procedure for finding an optimal Kalman gain.

To estimate the autocovariance based on data $\{y_k\}_{k=0}^{N}$, we use the following equation

$$\hat{\mathcal{C}}_j = \frac{1}{N_d - j} \sum_{i=1}^{N_d - j} \mathcal{Y}_{i+j} \mathcal{Y}_i{}^T \tag{6}$$

The complete autocovariance matrix R is then constructed by collecting these estimates for different lags into a single matrix. Assuming we want to estimate autocovariances up to lag M, the matrix R can be written as

$$R = \begin{bmatrix} \hat{\mathcal{C}}_0 & \hat{\mathcal{C}}_1 & \hat{\mathcal{C}}_2 & \cdots & \hat{\mathcal{C}}_M \\ \hat{\mathcal{C}}_1^T & \hat{\mathcal{C}}_0 & \hat{\mathcal{C}}_1 & \cdots & \hat{\mathcal{C}}_{M-1} \\ \hat{\mathcal{C}}_2^T & \hat{\mathcal{C}}_1^T & \hat{\mathcal{C}}_0 & \cdots & \hat{\mathcal{C}}_{M-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \hat{\mathcal{C}}_M^T & \hat{\mathcal{C}}_{M-1}^T & \hat{\mathcal{C}}_{M-2}^T & \cdots & \hat{\mathcal{C}}_0 \end{bmatrix} \tag{7}$$

where $\hat{\mathcal{C}}_0$ is the variance (or covariance for zero lag) and each $\hat{\mathcal{C}}_j$ (for $j > 0$) is the autocovariance at lag j. This matrix R is symmetric, with $\hat{\mathcal{C}}_j$ forming the off-diagonal entries and their transposes $\hat{\mathcal{C}}_j^T$ appearing symmetrically. Here, M is the number of lags.

The objective that is minimized to find an optimal Kalman gain is

$$\min_X \frac{1}{2} ||A_{LS} vec(X) - \mathcal{R}(N)|| \tag{8}$$

This objective is minimized to account for noisy measurements, enabling us to determine the process and measurement noise covariances, represented by X. The result of this minimization will provide estimates for $\hat{Q}_w$ and $\hat{R}_v$, which are contained in X. X minimizes the noise covariance.

The estimated covariances are

$$\hat{Q}_w = 0.4926 \tag{9}$$

$$\hat{R}_v = 0.1028 \tag{10}$$

The tuned Kalman filter is validated by applying it to the data given in Figure 2. The residual vector is given in Figure 5 and the autocovariance is illustrated in Figure 6.

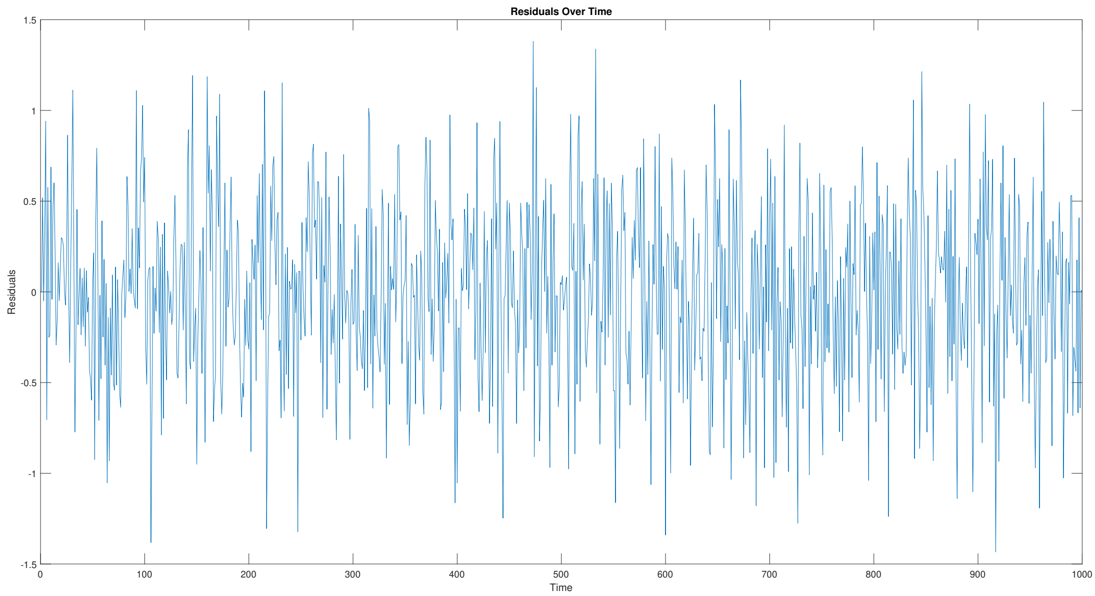

Figure 5: Residual of tuned Kalman filter.

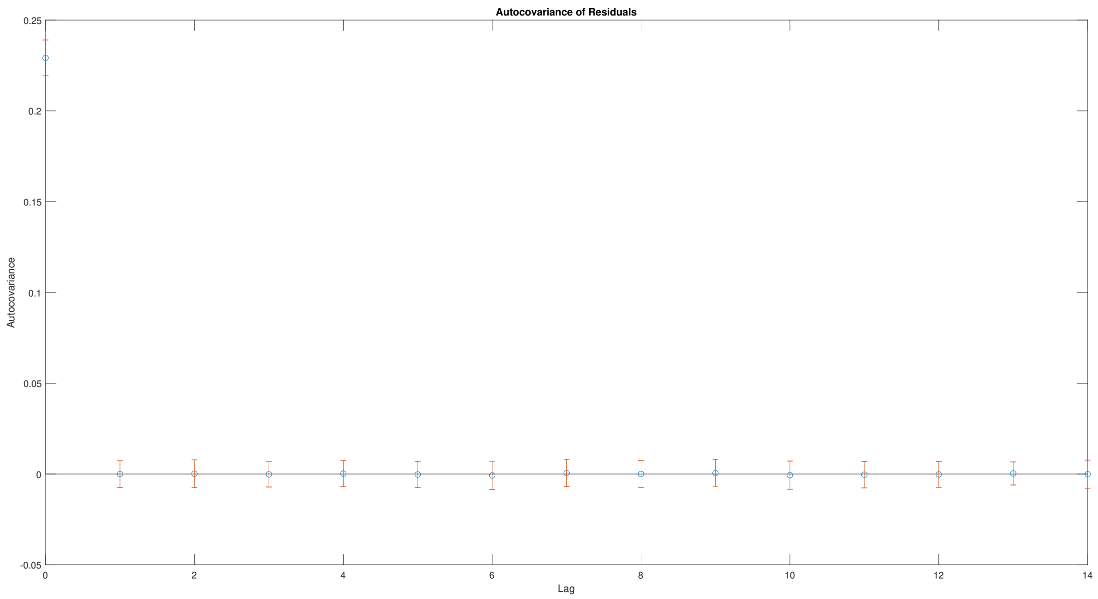

Figure 6: Autocovariance of tuned Kalman filter with lag up to 15 samples.