Hand-In Exercise: Admittance Controller

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1 Admittance Control

Design admittance controllers in operational space with the orientational part expressed a) with quaternion and b) with Euler angles.

1.1 Control Law

This section presents the admittance controller, where the translational part of the admittance controller is given by

$$M_p \Delta \ddot{p}_{cd} + D_p \Delta \dot{p}_{cd} + K_p \Delta p_{cd} = f \tag{1}$$

where M_p, D_p, K_p are 3×3 matrices, $f \in \mathbb{R}^3$ is force given in Base frame, and $\Delta p_{cd} = p_c - p_d$.

The rotational part of the admittance controller can be expressed in different ways depending on the representation use for orientation. Two representations are

$$M_o \Delta \dot{\omega}^d_{cd} + D_o \Delta \omega^d_{cd} + K'_o \epsilon^d_{cd} = \mu^d \tag{2}$$

$$M_o \Delta \ddot{\phi}_{cd} + D_o \Delta \dot{\phi}_{cd} + K_o \Delta \phi_{cd} = T^T(\phi_c)\mu \tag{3}$$

where M_o, D_o, K_o are 3×3 matrices, $\mu^d \in \mathbb{R}^3$ is the torque applied to the end-effector given in desired frame, $\Delta\omega_{cd} = \omega_c - \omega_d$, $\epsilon^d_{cd} = \eta_d \epsilon_c - \eta_c \epsilon_d - S(\epsilon_c)\epsilon_d$, and $\Delta\phi_{cd} = \phi_c - \phi_d$. The rotational stiffness matrix is

$$K'_o = 2E^T(\eta_{cd}, \epsilon^d_{cd})K_o$$

where K_o is the stiffness in Euler angle representation and

$$E(\eta, \epsilon) = \eta I - S(\epsilon)$$

1.2 Gain Selection

Write how the gains should be selected to obtain a critically damped system.

The gains should be selected in such a way that the M_p, D_p, K_p follow the equation:

$$\zeta = \frac{K_D}{2\sqrt{M_p K_p}} \tag{4}$$

$$D_p = 2\sqrt{M_p K_p} \tag{5}$$

where ζ is the damping ratio, K_d is the derivative gain, M_p is the mass, K_p is the proportional gain, and D_p is the damping gain. For a system to be critically damped, the damping ratio ζ should be equal to 1.

1.3 Implementation

Provide details on the implementation of the controller.

The moment in the desired frame is computed from the wrench h_e as

$$\mu^d = R_d^e(t)^T \cdot \mu_e + R_d^e(t)^T \cdot S(p_d^e) \cdot f_e \tag{6}$$

where $h_e = \begin{bmatrix} \mu_e \\ f_e \end{bmatrix}$ is an external wrench applied to the end-effector, $R_d^e(t)$ is the rotation matrix between the end-effector and the desired frame, and p_d^e is the position of the desired point relative to the end-effector frame.

1.3.1 Quaternion-Based Controller

The quaternion $(\eta_{cd}, \epsilon_{cd}^d)$ is obtained from integration of ω_{cd}^d as

$$q(t+dt) = \exp\left(\frac{dt}{2}\boldsymbol{\omega}_{cd}^d(t+dt)\right) * q(t) \tag{7}$$

The compliant frame p_c is obtained from the quaternion $(\eta_{cd}, \epsilon_{cd}^d)$ as

$$p_c = q_d \cdot q_{cd}^d \tag{8}$$

1.3.2 Euler Angle-Based Controller

The Euler angle $\Delta\phi_{cd}$ is obtained from integration of $\Delta\dot{\phi}_{cd}$ as

$$\Delta\phi_{cd} = \int \Delta\dot{\phi}_{cd}\, dt \tag{9}$$

The compliant frame p_c is obtained from the Euler angle $\Delta\phi_{cd}$ as

$$p_c = R_z(\Delta\phi_z) \cdot R_y(\Delta\phi_y) \cdot R_z(\Delta\phi_z) \cdot p \tag{10}$$

where $\Delta\phi_{cd} = (\Delta\phi_z, \Delta\phi_y, \Delta\phi_z)$

2 Simulation

Insert simulation results for the admittance controlled robot. The admittance controller should have a desired motion of your choice. In addition, no external force should be applied for the first five seconds, then a force $f = (1, 2, 3)$ N should be added for five seconds, then no external force for five seconds, then a torque $\mu = (1, 0.5, 1)$ Nm for five seconds and lastly no external force for five seconds. You should include figures that documents the simulation including applied wrench, desired motion, actual motion.

The desired motion is:

$$\begin{aligned}
x_{\text{ref}}(t) &= 0.3\sin\left(\frac{t}{3}\right), \\
y_{\text{ref}}(t) &= 0.3\sin\left(\frac{t}{3}\right)\cos\left(\frac{t}{3}\right), \\
z_{\text{ref}}(t) &= 0.1\sin(t), \\
q_{\text{ref}}(t) &= 1,
\end{aligned}$$

and is plotted on Figure 4.

The robot should be controlled in the following compliance frame:

$$T_c^s = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0.1 \\ 0 & 0 & 1 & 0.2 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

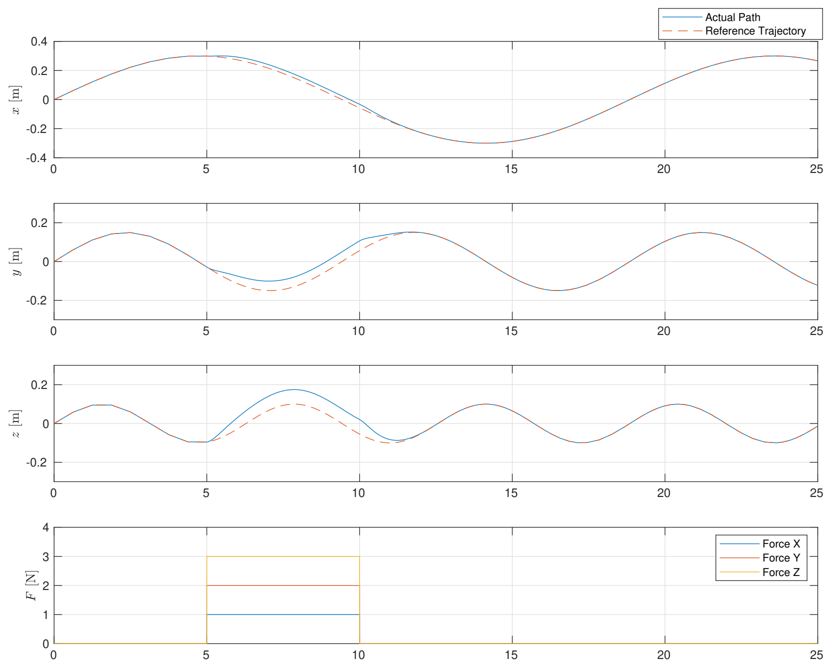

Figure 1: Desired motion, actual motion and applied forces vs time

where K_p=40, M=5 kg, initial pose and velocity are 0.

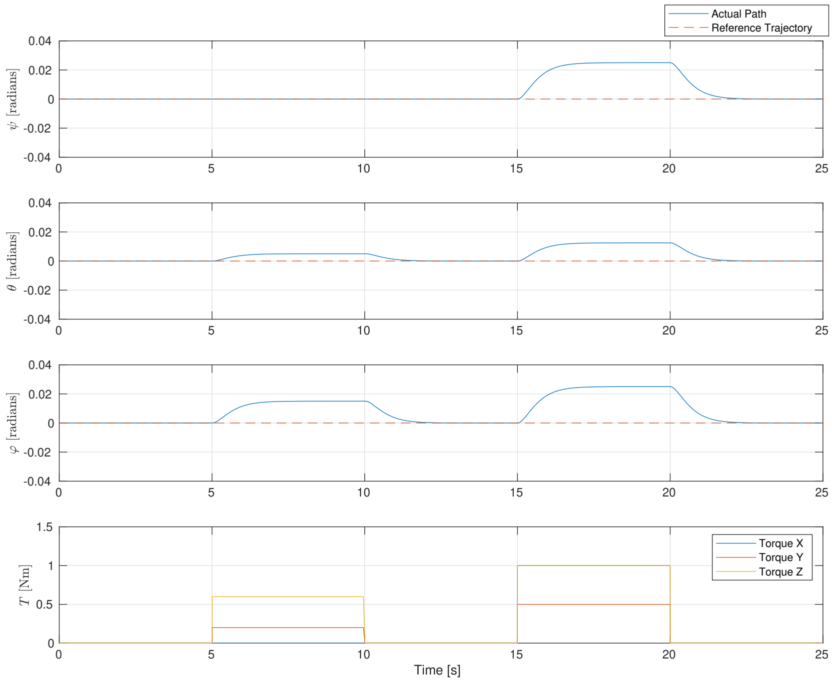

Figure 2: Desired angular motion, actual angular motion in Euler angles and applied torques vs time

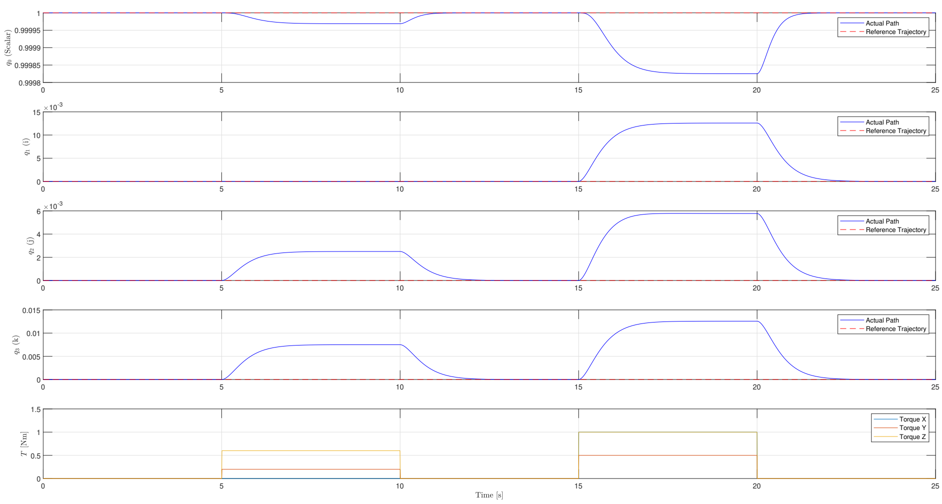

Figure 3: Desired angular motion, actual angular motion in quaternions and applied torques vs time

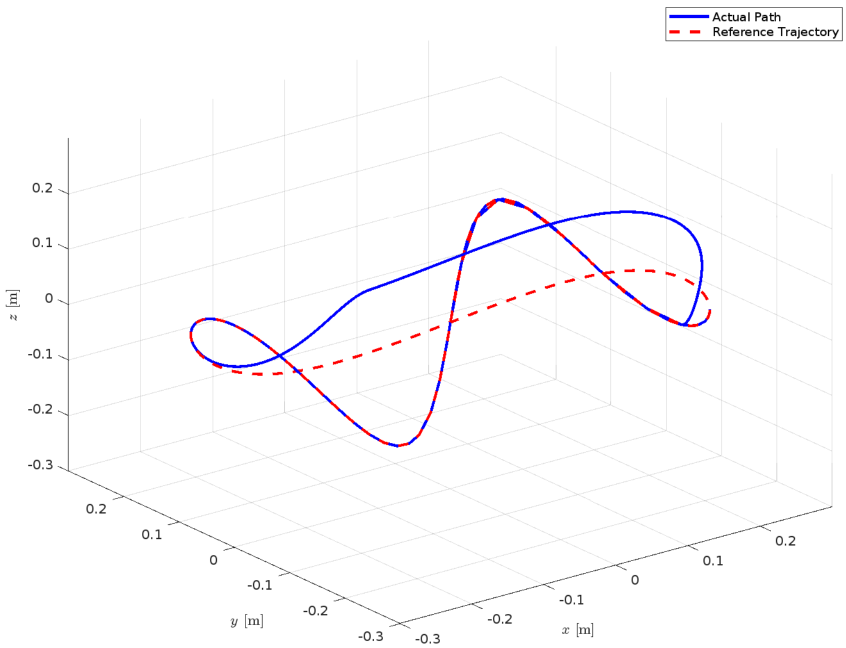

Figure 4: 3D representation of the translational motion of the robot